Dist. A

AD-A285 623

ATION PAGE

Form Approved
OMB No 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	July 20, 1994	Final Technical 1/15/90 to 1/14/94

4. TITLE AND SUBTITLE

Control of Complex Multibody Spacecraft (U)

5. FUNDING NUMBERS

AFOSR URI Grant
AFOSR-90-0105

6. AUTHOR(S)

P.S. Krishnaprasad

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Institute for Systems Research
University of Maryland
College Park, MD 20742

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force of Scientific Research
Bolling Air Force Base
Building 410
Washington, D.C. 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

AFOSR-TR- 94 0488

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

APPROVED FOR PUBLIC RELEASE:
DISTRIBUTION UNLIMITED

A

12b. DISTRIBUTION CODE

UL

13. ABSTRACT (Maximum 200 words)

The Project C-MULTICS (Control of Complex Multibody Spacecraft) is a center of excellence at the University of Maryland. The work supported by this project is concerned with the modeling, analysis, control and simulation of large scale complex multibody spacecraft with rigid and flexible components.

DTIC
ELECTE
OCT 2 0 1994
S D
F

DTIC QUALITY INSPECTED 2

14. SUBJECT TERMS	15. NUMBER OF PAGES
	32
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	

INSTITUTE FOR SYSTEMS RESEARCH

University of Maryland

College Park, MD 20742

Final Technical Report
January 15, 1990 - January 14, 1994

Control of Complex Multibody Spacecraft
AFOSR - URI Grant No. AFOSR-90-0105

Accesion For	
NTIS CRA&I	☑
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and / or Special
A-1	

Principal Investigator:

Professor P.S. Krishnaprasad
Department of Electrical Engineering
& Institute for Systems Research
University of Maryland
College Park, MD 20742

Co-principal Investigators:

Drs. Eyad Abed, Stuart Antman, John Baras, Carlos Berenstein, John Maddocks, Juan Simo.

94-32604

Control of Complex Multibody Spacecraft

(Project C-MULTICS)

AFOSR URI Grant AFOSR-90-0105

1. Mission and Personnel:

The study of complex, interconnected mechanical systems with rigid and flexible articulated components has drawn considerable attention from engineers and mathematicians. The problems of modeling, dynamic analysis, design and control synthesis for such systems offer formidable challenges of a cross-disciplinary nature. The exploitation of new types of actuation and sensing modes based on smart materials is of great current interest. Key technical motivations in this area have originated in aerospace engineering problems related to complex multibody spacecraft, the design of mechanisms with aerospace and other, possibly manufacturing-related, applications, and more recently in robotics (as in the control of space based robots and multifingered hands). To address some of these challenges, Project C-MULTICS was established in 1986 at the University of Maryland as a Center of Excellence under the auspices of the URI Program of the AFOSR. The continuation award in 1990 enabled the project team to boldly explore new and difficult scientific challenges as recognized from the first period of the award. The project has operated within the Systems Research Center (recently renamed the Institute for Systems Research (ISR)) at the University of Maryland, College Park. The period 93-94 was a no-cost extension period on this project.

Investigating such topics as modeling, dynamic analysis, design and control synthesis for such systems requires substantial mathematical tools and offers formidable challenges of a cross-disciplinary nature. The project consequently involved researchers from several areas, including P. S. Krishnaprasad, a professor in the ISR and the Electrical Engineering Department; Eyad Abed and John Baras, both of ISR and Electrical Engineering; Carlos Berenstein of the ISR and the Mathematics Department; Stuart Antman and John Maddocks from the Mathematics Department; and Juan Simo of Stanford University's Division of Applied Mechanics. The project team members also collaborate with Professor Jerrold Marsden of the Mathematics Department of the University of California at Berkeley, Professor Tudor Ratiu of the Mathematics Department at UC Santa Cruz, and Professor Anthony Bloch of the Mathematics Department at Ohio State University.

The project was organized into three topical areas (as originally proposed): (1) Analytical mechanics and modeling; (2) Nonlinear and distributed control; and (3) Advanced computation research. The objectives of the project within these areas included investigation of, basic mechanics and hamiltonian and dissipative structures in coupled three-dimensional multibody systems with flexible and fluid components, orbit-attitude coupling

and its impact on stability of relative equilibria, the role of constraints in mechanical systems and geometric modeling of constraints; the development of nonlinear and distributed controls using the detailed understanding of nonlinear dynamics gained in the project; the use of smart materials to implement various control techniques and to explore in the laboratory a variety of phenomena related to actuator properties and their impact on control performance; and the development of high-fidelity numerical integration algorithms and simulation tools and software tools to support the analysis and design of complex multibody systems.

A significant body of the results that have been obtained in this project continue to have applications to the solution of problems arising from control-structure interactions, as in spacecraft with large flexible components. Radar arrays, large earth-observation platforms, truss-mounted articulated systems, space telescopes, tethered systems are key examples. In addition, it is expected that the results of this project will lead to design of spacecraft and booster systems to accomodate fluid-structure interaction as in liquid-slosh in fuel containers.

2. Executive Summary of Accomplishments during 1990-1994:

1. One of the major strengths of the project team has been its past accomplishments in geometric mechanics in the context of multibody systems. During 1990-94, the team managed to build on these accomplishments as well as generate exciting new results. A key advance is the new synthesis of results in geometric mechanics and geometric control theory brought about by the extension of the energy- momentum method to gyroscopic systems. Gyroscopic stabilization of large multibody systems is achievable through the techniques presented in the work of L-S. Wang and P.S. Krishnaprasad (*Journal of Nonlinear Science*, vol. 2, 1992, pp. 367-415). The method of amended potentials is used as a tool for control synthesis and illustrated through key examples of complex multibody systems.

2. Closely related to this work on gyroscopic stabilization is the work of Bloch, Krishnaprasad, Marsden and Alvarez (*Automatica*, vol. 28, no. 4, 1992, pp. 745-756). In addition to the exploitation of nonlinear feedback, this work identifies a valuable role for the use of geometric phases to compensate accurately for the drifts in satellite attitude arising from external disturbances.

3. The work of L-S. Wang, P.S. Krishnaprasad and J. Maddocks on the dynamics of rigid bodies (e.g. LEO satellites) in central force fields has led to new results on poisson structure, precise computation of oblique relative equilibria, stability assessment for such equilibria, and bifurcation studies (*Celestial Mechanics and Dynamical Astronomy*, vol. 50, 1991, pp. 349-386, and *Journal of Astronautical Sciences*, vol. 40, no. 4, 1992, pp. 449-478. These results are of interest to JPL as a possible means to find unusual passively stable attitude for large earth observation platforms.

4. One of the most challenging problems in large earth-observation spacecraft arises from the 'day-night' thermal cycling. As the satellite alternates between illumination by the sun and the darkness due to earth-occlusion, there are significant temperature changes in the flexible components of the spacecraft. These periodic changes force thermo-elastically driven vibrations which in turn cause attitude drift. A model problem of this type is the

one on Figure 1 representing a rigid body with two driven mass-spring oscillators. This problem is analyzed carefully in the paper (Contol Problems on Principal Bundles and Nonholonomic Mechanics by R. Yang and P. S. Krishnaprasad, in *Proceedings of 1991 IEEE Conference on Decision and Control*, pp. 1133-1138, IEEE) and in the Ph.D. thesis of Yang where some closely related optimal control problems are explored and new drift formulas presented.

5. Dissipation can induce instabilities. A powerful new theorem along these lines generalizing the classical work of Kelvin and Cetaev to relative equilibria has been proved by Bloch, Krishnaprasad, Marsden and Ratiu (*Annales de'l Institut Henri Poincare: analyse non-lineaire*, vol. 11, no. 1, 1994, pp. 37-90). This result is expected to have very wide applications including the liquid-slosh problem referred to above. A finite dimensional example shows the possibility of instabilities due to actuator failure in a three axis stabilized satellite.

6. The result in (5) above has been extended to include a new class of dissipation mechanisms known as *double bracket dissipation.* This work further allows one to consider dissipation in the so-called Arnold block associated to the block-diagonalization theorem of geometric mechanics. This work is under review for journal publication and is available as a technical report distributed by the Fields Institute (Canada). The results here also allow one to consider new dissipation mechanisms in fluids.

7. A new experimental program in smart actuators has been initiated, to explore the role of terfenol, a magneto-elastic material, in structural control. A new terfenol-based rotory actuator with high torque output is under study. Piezo-ceramic material based actuators have been used in the lab in the study of structural control via distributed actuation. Applications to system identification have been made using such actuators. Considerable success has been achieved in accurate modeling and compensation for friction in bearings of the type used in scan platforms. Adaptive control strategies have been shown to be effective.

8. A new study on impact in flexible structures has been carried out. New impact force computation algorithms and theorems have been established. A type of robust control theory in the presence of impact disturbances is under study. Experimental confirmation of force control strategies has been done.

9. Reliable long-term numerical simulation to validate complex designs is now feasible due to the results on symplectic and Poisson integration generated by Project C-MULTICS. For the first time, a unified computational framework for rods, plates and shells with appropriate geometric properties has been created. A distributed, object oriented, computational environment supporting mixed symbolic and numeric computation and 3-D graphical visualization has been developed. This facility serves as a test-bed and experimental tool for the study of multibody systems. Many key software innovations are demonstrated in this facility.

10. M. Lanza and S. S. Antman have analyzed several steady-state problems of flow past nonlinearly elastic shells. There is an extensive engineering literature on fluid-solid interactions, encompassing extremely interesting dynamical problems,much of which based

on the use of ad hoc models for the force field exerted by the fluid on the deformable solid. One of their goals was to treat these interactions honestly. Specifically, they studied steady irrotational flow of inviscid fluids past a closed cylindrical shell, past a smooth boundary that is rigid except for an embedded deformable panel, past a panel behind which there is a wake, and past a closed axisymmetric shell. The large deformation of these shells is described by a general geometrically exact theory that accounts for flexure, extension, and shear and allows a very general class of nonlinear constitutive relations.

The fluid-solid interaction consists in the fact that the shape of the shell determines the flow, and the flow determines the shape of the shell. This observation leads to an effective strategy for analyzing these problems. The analysis presents some serious technical difficulties that do not arise in the flow past a rigid obstacle. The products of these studies are demonstrations of how solutions depend on the velocity and pressure at infinity. This work may have direct applications to problems in transatmospheric vehicles that are of interest to the Air Force.

11. Bifurcation control tools developed as part of Project C-MULTICS have been found to be very relevant to stall avoidance and stall mitigation problems for high performance aircraft. This includes issues of aircraft dynamic stall as well as stall of axial-flow-compressor turbine jet engines. The jet engine work has resulted in an understanding, for the first time, of the nonlinear mechanism behind compressor stall. An important anticipated benefit of these results is a set of techniques allowing one to control the jet engine at the onset of stall, and to control the severity of the stall that occurs. This work has been conducted with the active collaboration of research engineers of the General Electric Company, from both the Corporate Research and Development Center (Schenectady, NY) and the Aircraft Engines Business Group (Evendale, OH). GE has provided compressor test data (for instance, for the F101 engine) and was involved in problem definition as well as interpretation of the results.

12. The work on geometric phases and related control theory has led to fundamental understanding of nonholonomic mechanics from a geometric viewpoint. New reduction theorems have been proved using geometric techniques. These insights have also shed new light on the problem of constructive controllability of nonholonomic systems. A special focus on periodic control was motivated by the fact that there are many interesting classes of nonholonomic problems for which there do not exist smooth time-invariant stabilizing feedback laws, but there exist time-periodic stabilizing feedback laws. Some of these systems live on Lie groups. New averaging theorems for left-invariant systems on Lie groups have been obtained. These have led to constructive control techniques for attitude control, and other motion control problems in Lie groups. New results on nonholonomic motion planning that integrate nonlinear dynamics with a motion control language have been obtained.

13. There have been significant contacts with personnel at Air Force labs and at other labs where work of interest to the Air Force goes on. Typically these contacts included lab visits, discussions or presentations of our project work, followed by suggestions for problem directions and sharing of reports (papers and theses). These contacts include the vibrations group at Phillips Lab (Edward AFB), flight controls group at Eglin AFB,

structures and dynamics group at Wright Patterson AFB and JPL. In addition, more recently we have had contacts with the materials laboratory at Wright to explore possible uses of our work in process control problems involving distributed parameter systems and nonlinear phenomena. There is a specific application of interest namely, chemical vapor deposition of metal oxides on sapphire fibers to be used as re-inforcement for aircraft engine composites.

The following list contains works that have been published, completed, or nearly completed during the grant period. In the next few sections we will discuss selected accomplishments reported in these papers/reports/theses, and will also discuss how the work of various investigators is tied together.

[1] L. Saydy, A.L. Tits and E.H. Abed, "Guardian maps and the generalized stability of parametrized families of matrices and polynomials," *Mathematics of Control, Signals and Systems*, Vol. 3, No. 4, pp. 345-371, 1990.

[2] E.H. Abed and H.-C. Lee, "Nonlinear stabilization of high angle-of-attack flight dynamics using bifurcation control," *Proc. 1990 American Control Conference*, American Automatic Controls Council, pp. 2235-2238.

[3] D.-C. Liaw and E.H. Abed, "On nonlinear station-keeping control of tethered satellites," *Proc. 1990 American Control Conference*, American Automatic Controls Council, p. 2445.

[4] D.-C. Liaw, *Feedback Stabilization via Center Manifold Reduction with Application to Tethered Satellites*, Ph.D. Dissertation, Dept. of Electrical Engineering, University of Maryland, August 1990, (Advisor: E.H. Abed).

[5] J.-H. Fu and E.H. Abed, "Families of Liapunov functions for nonlinear systems in critical cases," *Proc. 29th IEEE Conf. on Decision and Control*, IEEE, 1990, pp. 1300-1305.

[6] D.-C. Liaw and E.H. Abed, "Stabilization of nonlinear systems via center manifold reduction," *Proc. 29th IEEE Conf. on Decision and Control*, IEEE, 1990, pp. 804-809.

[7] D.-C. Liaw and E.H. Abed, "Stabilization of tethered satellites during station keeping," *IEEE Trans. Automatic Control*, Vol. AC-35, No. 11, pp. 1186-1196, November 1990.

[8] L. Saydy, A.L. Tits and E.H. Abed, "Maximal range for generalized stability - Application to physically motivated examples," *International J. Control*, Vol. 53, No. 4, pp. 837-846, April 1991.

[9] E.H. Abed, J.-H. Fu, H.-C. Lee and D.-C. Liaw, "Bifurcation control of nonlinear systems," in *New Trends in System Theory*, G. Conte, A.M. Perdon and B. Wyman, Eds., pp. 35-41, Boston: Birkhäuser, 1991.

[10] H.-C. Lee and E.H. Abed, "Washout filters in the bifurcation control of high alpha flight dynamics," *Proc. 1991 American Control Conference*, American Automatic Controls Council, pp. 206-211.

[11] H.-C. Lee, *Robust Control of Bifurcating Nonlinear Systems with Applications*, Ph.D. Dissertation, Dept. of Electrical Engineering, University of Maryland, August 1991, (Advisor: E.H. Abed).

[12] H. Wang and E.H. Abed, "Bifurcation control of chaotic dynamical systems," *Proceedings of NOLCOS'92: Nonlinear Control System Design Symposium*, (M. Fliess, Ed.), June 24-26, 1992, Bordeaux, France, pp. 57-62; Published by the International Federation of Automatic Control.

[13] D.-C. Liaw and E.H. Abed, "Analysis and control of rotating stall," *Proc. NOLCOS'92: Nonlinear Control System Design Symposium*, (M. Fliess, Ed.), June 24-26, 1992, Bordeaux, France, pp. 88-93; Published by the International Federation of Automatic Control.

[14] H. Wang, E.H. Abed and H.-C. Lee , "Robust control of a chaotic dynamical system," *Proc. 1992 American Control Conference*, American Automatic Controls Council, pp. 2236-2237.

[15] H. Wang and E.H. Abed, "Controlling chaos and targeting in a thermal convection loop," *Proc. Second IFAC Workshop on System Structure and Control*, Sept. 3-5, 1992, Prague, Czechoslovakia, pp. 494-497 (invited); Published by the International Federation of Automatic Control.

[16] E.H. Abed, H.O. Wang and R. Chen, "Stabilization of period doubling bifurcations and implications for control of chaos," *Proc. 31st IEEE Conference on Decision and Control*, 1992, IEEE, pp. 2119-2124.

[17] J.-H. Fu and E.H. Abed, "Families of Lyapunov functions for nonlinear systems in critical cases," *IEEE Trans. Automatic Control*, Vol. 38, No. 1, pp. 3-16, Jan. 1993.

[18] E.H. Abed, P.K. Houpt and W.M. Hosny, "Bifurcation analysis of surge and rotating stall in axial flow compressors," *ASME Journal of Turbomachinery*, Vol. 115, No. 4, pp. 817-824, Oct. 1993.

[19] J.-H. Fu and E.H. Abed, "Linear feedback stabilization of nonlinear systems with an uncontrollable critical mode," *Automatica*, Vol. 29, No. 4, pp. 999-1010, July 1993.

[20] H. O. Wang, *Control of Bifurcations and Routes to Chaos in Dynamical Systems*, Ph.D. Dissertation (Inst. Syst. Res. Thesis Rept. No. 93-10), Dept. of Electrical Engineering, University of Maryland: December 1993, (Advisor: E.H. Abed).

[21] E.H. Abed and H.O. Wang, "Feedback Control of Bifurcation and Chaos in Dynamical Systems," invited chapter for *Recent Developments in Stochastic and Nonlinear Dynamics: Applications to Mechanical Systems*, a volume commemorating the sixtieth birthday of Professor S.T. Ariaratnam, CRC Press, to be published in 1994,

W. Kliemann and N. Sri Namachchivaya, Editors. Also, ISR Tech. Rept. No. TR 93-74.

[22] D.-C. Liaw and E.H. Abed, "Active Control of Compressor Stall Inception: A Bifurcation-Theoretic Approach," *Automatica*, accepted, 1994.

[23] H.O. Wang and E.H. Abed, "Bifurcation Control of a Chaotic System," Automatica, revised for publication, 1994.

[24] E.H. Abed, H.O. Wang and R.C. Chen, "Stabilization of period doubling bifurcations and implications for control of chaos," *Physica D*, Vol. 70, Nos. 1-2, pp. 154-164, January 1994.

[25] S.S. Antman and P.V. Negron-Marrero, "Singular Global Buckling Problems for Anisotropic Plates," *Proceedings of the Royal Society of London*, vol 427, 1990, pp. 95–137.

[26] S.S. Antman and J.F. Pierce, "The Intricate Global Structure of Buckled States of Compressible Columns," *SIAM Journal on Applied Mathematics*, vol. 50, 1990, pp. 395–419.

[27] S.S. Antman, "Global Properties of Buckled States of Plates That Can Suffer Thickness Changes," *Archive for Rational Mechanics and Analysis* vol. 110, (1990), pp. 103–117.

[28] S.S. Antman and M. Lanza de Cristoforis, "The Large Deformation of Nonlinearly Elastic Tubes in Two-Dimensional Flows," *SIAM Journal of Mathematical Analysis*, vol. 22, 1991, pp. 1193–1221.

[29] S.S. Antman and M. Lanza de Cristoforis, "Cavitational Flow past a Nonlinearly Elastic Panel," *Indiana University Mathematics Journal*, vol. 39, 1990, pp. 383-412.

[30] S.S. Antman and M. Lanza de Cristoforis, "The Large Axisymmetric Deformation of a Shell in a Perfect Flow," *Annales de l'Institut Henri Poincaré, Analyse non linéaire*, vol. 9, 1992, pp. 433-464.

[31] S.S. Antman and R.S. Marlow, "Transcritical Buckling of Columns," *Zeitschrift für angewandte Mathematik und Physik*, vol. 43, 1992, pp. 7-27.

[32] S.S. Antman and R.S. Marlow, "Material Constraints, Lagrange Multipliers, and Compatibility, Applications to Rod and Shell Theories," *Archive for Rational Mechanics and Analysis*, vol. 116, 1991, pp. 257-299.

[33] S.S. Antman and M. Lanza de Cristoforis, "Nonlinear, Nonlocal Problems of Fluid-Solid Interactions," in *Degenerate Diffusion*, edited by W.-M. Ni, L.A. Peletier, and J.L. Vazquez, IMA Volumes in Mathematics and its Applications, Springer-Verlag, New York, 1993, pp. 1-18.

[34] S.S. Antman and M. Lanza de Cristoforis, "Perfect Flow Past a Nonlinearly Elastic Panel," to appear in *Nonlinear Partial Differential Equations and their Applications, Collège de France Seminar*, edited by H. Brezis and J.-L. Lions, Longmans, 1994.

[35] S.S. Antman and R.S. Marlow, "New Phenomena in the Buckling of Arches described by Refined Models," *International Journal of Solids and Structures*, vol. 30, 1993, pp. 2213–2241.

[36] S.S. Antman, "Continuation Methods in Nonlinear Elasticity," to appear in *Proceedings of World Congress of Nonlinear Analysts*, De Gruyter.

[37] S.S. Antman and T.I. Seidman, "Quasilinear Hyperbolic-Parabolic Equations of Nonlinear Viscoelasticity," *Journal of Differential Equations*, to appear, 44 pages.

[38] S.S. Antman and R.S. Marlow, "Large Motions of Light Flexible Bodies," virtually complete.

[39] S.S. Antman, "Large Motions of Light Viscoelastic Rods," *Proceedings of the Second International Congress on Nonlinear Mechanics, Beijing*, edited by W.-Z. Chien, Peking University Press, 1993, pp 1-8.

[40] S.S. Antman, *Nonlinear Problems of Elasticity*, Springer-Verlag, 1994 (700 pages).

[41] John Baras and Yan Zhuang, "Active Vibration Damping Using Smart Materials," Proc. Workshop on Distributed Parameter Modeling and Control of Flexible Aerospace Systems", Williamsburg, Virginia, June 8-10, 1992.

[42] Donald R. Augenstein, John Baras and Shalom M. Fisher, "Parameter Identification of Large Spacecraft Systems Based on Frequency Characteristics", Proc. 31st IEEE Conference on Decision and Control, 1992.

[43] M.E. Baraka, *Optimal Control of Large Space Structures*, Ph.D. Thesis, 1992 (Advisor: J.S. Baras) .

[44] D.R. Augenstein, *Parameter Identification of Large Spacecraft Systems, Based on the Frequency Characteristics of the Hankel Operator*, M.S. Thesis, 1992 (Advisor: J.S. Baras).

[45] B. Frankpitt, *A Frequency Domain Design for the Control of a Distributed Parameter System*, M.S. Thesis, 1993 (Advisors: J.S. Baras and C.A. Berenstein).

[46] C.A. Berenstein and A. Yger, "Bounds for the degrees in the Division Problem", *Mich. Math. J.* 37 (1990), 26–44.

[47] C.A. Berenstein, R. Gay and A. Yger, "Inversion of the Local Pompeiu Transform", *J. Analyse Math.* 54 (1990), 259–287.

[48] C.A. Berenstein and A. Yger, "Effective Bezout Estimates in $\mathbf{Q}[z_1, \ldots, z_n]$", *Acta Mathematica* 66 (1991), 69–120.

[49] C.A. Berenstein and A. Yger, "Bounds for the Degrees in Polynomial Equations", *Proc. Symp. Pure Math.* 52 (1991), part 1, 23–28.

[50] C.A. Berenstein and A. Yger, "Une Formule de Jacobi et ses Consequences", *Annales Scient. Ecole Normale Sup.* Paris 24 (1991), 363–377.

[51] C.A. Bernstein and A. Yger, "About Ehrenpreis' Fundamental Principle", *Geom. and Alg. Aspects in Several Complex Variables*, Berenstein and D. C. Struppa (ed.), Proc. Conf. in honor of Ehrenpreis' 60th birthday, EditEl 1991, 47–61.

[52] C.A. Berenstein and D.C. Struppa, "Recent Improvements in the Complexity of the Effective Nullstellensatz", *Linear Algebra and its Applications* 157 (1991), 203–215.

[53] C.A. Berenstein and A. Yger, "Formules de Representation Integrale et Problemes de Division", *Diophantine Approximation and Transcendental Numbers, Luminy 1990*, P. Philippon (ed.), Walter de Gruyter and Co., Berlin, 1992, 15–37.

[54] C.A. Berenstein, R. Gay, A. Vidras and A. Yger, "Residues and Bezout Identities", *Progress in Mathematics 114, Birkhäuser*, 1993, 158 pages.

[55] L.-S. Wang, P.S. Krishnaprasad, and J.H. Maddocks "Hamiltonian Dynamics of a Rigid Body in a Central Gravitational Field", *Celestial Mechanics and Dynamical Astronomy*, Vol. 50, pp. 349-386 , (1991).

[56] A.M. Bloch, P.S. Krishnaprasad, J.E. Marsden and G.S. Alvarez "Stabilization of Rigid Body Dynamics by Internal and External Torques", *Automatica*, vol. 28, no. 4, pp. 745-756, 1992.

[57] L-S. Wang and P.S. Krishnaprasad "Gyroscopic Control and Stabilization", *J. Nonlinear Science*, vol. 2, pp. 367-415, (1992).

[58] L-S. Wang, J.H. Maddocks, and P.S. Krishnaprasad "Steady Rigid-Body Motions in a Central Gravitational Field", *J. Astronautical Sciences*, vol. 40, no. 4, pp. 449-478, 1992.

[59] R. Shahidi, M. Shayman and P.S. Krishnaprasad, "Mobile Robot Navigation using Potential Functions", in *Proceedings of 1991 IEEE International Conference on Robotics and Automation*, IEEE, New York, pp 2047-2053.

[60] P.S. Krishnaprasad, and R. Yang, "Geometric Phases, Anholonomy and Optimal Movement" , in *Proceedings of 1991 IEEE International Conference on Robotics and Automation*, IEEE, New York, 1991, pp 2185-2189.

[61] M. Austin, P.S. Krishnaprasad and L-S. Wang, "On Symplectic and almost Poisson Integration of Rigid Body Systems" , invited paper for *International Conference on Computational Engineering Science*, Melbourne, Australia, August 1991.

[62] A.M. Bloch, P.S. Krishnaprasad, J.E.Marsden, and T.S. Ratiu, "Asymptotic Stability, Instability and Stabilization of Relative Equilibria" , invited paper in *Proceedings of 1991 American Control Conference*, American Automatic Control Council, Philadelphia.

[63] L-S.Wang, P.S. Krishnaprasad and W.P. Dayawansa, "Mechanical Systems with Partial Damping" (1991), in *Proceedings of* 30th *IEEE Conference on Decision and Control*, IEEE, New York, pp 2356-2360.

[64] P.S. Krishnaprasad, R. Yang, and W.P. Dayawansa, "Control Problems on Principal Bundles and Nonholonomic Mechanics" (1991), invited paper, in *Proceedings of 30th IEEE Conference on Decision and Control*, IEEE, New York, pp 1133-1138.

[65] Naomi Ehrich Leonard, *An Investigation of Control Strategies for Friction Compensation* , M.S. Thesis, Department of Electrical Engineering and Systems Research Center, August 1991, (Advisor: P.S. Krishnaprasad).

[66] Ioanis Salmatjidis, *Impact and Force Control of Flexible Maniuplators*, M.S. Thesis, Department of Electrical Engineering and Systems Research Center, August 1991, (Advisor: P.S. Krishnaprasad).

[67] W. Shadwick, P.S. Krishnaprasad and T.S. Ratiu, editors, *Mechanics Day: Celebration of the 25th Anniversary of the Publication of "Foundations of Mechanics"*, in press, American Math Society, (in the Fields Institute Series), Providence, R.I.

[68] M. Austin, P.S. Krishnaprasad and L-S. Wang, "Almost Poisson Integration of Rigid Body Systems" (1993) *J. Computational Physics*, Vol. 107, No. 1, pp 105-117.

[69] R. Yang and P.S. Krishnaprasad, "On the Geometry and Dynamics of Floating Four Bar Linkages", (1994) *Dynamics and Stability of Systems*, Vol 9, No. 1, pp 19-45.

[70] A.M. Bloch, P.S. Krishnaprasad, J.E. Marsden, T.S. Ratiu, "Dissipation Induced Instabilities", (1994) in *Annales de L'Institut Henri Poincare: Analyse Non Lineare*, Vol. 11, No. 1, pp 1-54.

[71] R. Yang, P.S. Krishnaprasad and W.P. Dayawansa, "Optimal Control of a Rigid Body with Two Oscillators", (1994), to appear in, *Fields Institute Communications*, also Institute for Systems Research Technical Report TR 93-63, 28 pages.

[72] A.M. Bloch, P.S. Krishnaprasad, J.E. Marsden and T.S. Ratiu, "The Euler-Poincare' Equations and Double Bracket Dissipation" (1994), submitted to *Communications. Math. Physics.*, also Fields Institute Technical Report, FI94-CT01, 33 pages.

[73] N.E. Leonard and P.S. Krishnaprasad, "Motion Control of Drift-Free, Left-Invariant Systems on Lie Groups Part I: Averaging and Controllability", (1994), submitted to *IEEE Trans. Aut. Control*, 31 pages.

[74] N.E. Leonard and P.S. Krishnaprasad "Motion Control of Drift-Free, Left-Invariant Systems on Lie Groups Part II: A General Constructive Control Algorithm", (1994), submitted to, *IEEE Trans. Aut. Control*, 38 pages.

[75] P.S. Krishnaprasad, "Optimal Control and Poisson Reduction", (1994), to be submitted, also Institute for Systems Research Technical Report TR 93-87, 16 pages.

[76] N.E. Leonard and P.S. Krishnaprasad, "Averaging on Lie Groups, Attitude Control and Drift" (1993), in *Proceedings of the Conference on Information Sciences and Systems*, The Johns Hopkins University, pp 369-37 .

[77] D. Tsakiris and P.S. Krishnaprasad, "Motion Planning for a Class of Active Robotic Systems", (1993), in *Proceedings of IEEE Symposium on New Directions in Control Theory and Applications*, June 21 - 23, Greece.

[78] R. Yang, P.S. Krishnaprasad and W. Dayawansa, "Chaplygin Dynamics and Lagrangian Reduction", (1993), in Chien W-Z., Guo Z-H., Guo Y-Z., eds., *Proceedings of 2nd International Conference on Nonlinear Mechanics* (ICNM II), Peking University Press, China, pp 745-749.

[79] N.E. Leonard and P.S. Krishnaprasad, "Averaging for Attitude Control and Motion Planning", (1993), in *Proceedings of 32nd IEEE Conference on Decision and Control*, IEEE, New York, pp 3098-3104.

[80] Q.F. Wei, W.P. Dayawansa and P.S. Krishnaprasad, "Modeling of Impact on a Flexible Beam", (1993), in *Proceedings of 32nd IEEE Conference on Decision and Control*, IEEE, New York, pp 1377-1382.

[81] R. Rezaiifar, Y.C. Pati, P.S. Krishnaprasad and W. P. Dayawansa, "Wavelet Based Identification of Smart Structures with Surface Mounted Actuators and Sensors", (1993), in *Proceedings of 32nd IEEE Conference on Decision and Control*, IEEE, New York, pp 486-491.

[82] T.K. Shing, L-W. Tsai and P.S. Krishnaprasad, "An Improved Model for the Dynamics of Spur Gear Systems with Backlash Consideration", (1993), *Advances in Design Automation* - vol. 1, DE-vol. G5-1, pp 235-243, ASME, New York.

[83] Y.C. Pati, R. Rezaiifar and P.S. Krishnaprasad, "Orthogonal Matching Pursuit: Recursive Function Approximation with Applications to Wavelet Decomposition", (1993), in *Proc. 27th Asilomar Conference on Signals, Systems and Computers*, Nov. 1-3, 1993.

[84] Y.C. Pati, R. Rezaiifar, P.S. Krishnaprasad and W.P. Dayawansa, "A Fast Recursive Algorithm for System Identification and Model Reduction using Rational Wavelets", (1993), in *Proc. 27th Asilomar Conference on Signals, Systems and Computers*, Nov. 1-3, 1993.

[85] N.E. Leonard and P.S. Krishnaprasad, "High-Order Averaging on Lie Groups and Control of an Autonomous Underwater Vehicle", (1994), to appear in *Proc. 1994 American Control Conference*, 28 pages.

[86] Q-F. Wei, W.P. Dayawansa and P.S. Krishnaprasad, "Approximation of Dynamical Effects due to Impact on Flexible Bodies", (1994), to appear in *Proc. 1994 American Control Conference*, 13 pages.

[87] P.S. Krishnaprasad and D.P. Tsakiris, "Nonholonomic Variable Geometry Truss Assemblies I: Motion Control", (1994), invited paper for *IFAC Symposium on Robot Control '94*, Capri, Italy, Sept 19-21, (also Institute for Systems Research Technical Report TR 93-90) 32 pages.

[88] N.E. Leonard and P.S. Krishnaprasad, "Control of Switched Electrical Networks Using Averaging on Lie Groups", (1994), to appear in, *Proceedings 33rd IEEE Conference on Decision and Control*, 26 pages.

[89] P.S. Krishnaprasad and Dimitris Tsakiris, "G-Snakes: Nonholonomic Kinematic Chains on Lie Groups", (1994), to appear in, *Proceedings 33rd IEEE Conference on Decision and Control.*

[90] N.E. Leonard and P.S. Krishnaprasad, "Averaging and Motion Control on Lie Groups", (1993), Institute for Systems Research Technical Report TR 93-59, 47 pages.

[91] N.E. Leonard, *Averaging and Motion Control of Systems on Lie Groups*, (1994), Ph.D. dissertation, Department of Electrical Engineering, University of Maryland, College Park, (Advisor: P.S. Krishnaprasad).

[92] Y. Ozkazanc, *Dynamics and Stability of Spacecraft with Fluid-filled Containers*, (1994), Ph.D. dissertation, Department of Electrical Engineering, University of Maryland, College park, (Advisor: P.S. Krishnaprasad).

[93] John H. Maddocks "On The Stability of Relative Equilibria", *IMA J. Applied Math.*, 46, 1991, pp. 71-99.

[94] John H. Maddocks "On Second-Order Conditions In Constrained Variational Principles", to appear, *J. Opt. Thy. and Appl.*

[95] John H. Maddocks & Robert L. Sachs "On The Stability Of KdV Multi-Solitons", submitted to, *Comm. Pure and Applied Math.*.

[96] John H. Maddocks & Reza Malek-Madani "Steady-State Shear-Bands In Thermo-Plasticity, Part I: vanishing yield stress", to appear, *Int. J. Solids and Structures.*

[97] John H. Maddocks and D. Yakobson "On the Dynamics of Chains", to appear, *SIAM J. Applied Math.*

[98] J.C. Alexander & J.H. Maddocks, "Bounds On The Friction Dominated Motion Of A Pushed Object", accepted for publication, *Int. J. Robotics Research.*

[99] J.H. Maddocks and D.J. Dichmann, "Conservation Laws in the Dynamics of Rods", *preprint 1992*, University of Maryland, Department of Mathematics.

[100] J.H. Maddocks and R.L. Sachs, "Constrained Variational Principles and Stability in hamiltonian Systems" *to appear in an IMA Volums in Mathematics and It Applications* 'Hamiltonian Dynamical Systems' Eds. Dumas, Meyer and Schmidt, 1994.

[101] J. H. Maddocks and R.L. Pego, "An Unconstrained Hamiltonian Formulation for Incompressible Fluid Flow" *preprint 1994*, University of Maryland, Department of Mathematics.

[102] J.H. Maddocks and M.L. Overton, "Stability Theory for Dissipatively Perturbed Hamiltonian Systems", 1994, Submitted for publication in *Comm. Pure Appl. Math.*

[103] J.C. Simo, M.S. Rifai & D.D. Fox "On A Stress Resultant Geometrically Exact Shell Model. Part VI: Conserving Algorithms For Nonlinear Dynamics", preprint, *Division of Applied Mehcanics, Department of Mechanical Engineering, Stanford University,*

[104] J.C. Simo "The (Symmetric) Hessian For Geometrically Nonlinear Models In Solid Mechanics: Intrinsic Definition And Geometric Interpretation", *to appear, Comp. Meth. Appl. Mech. Eng.*

[105] J.C. Simo & K.K. Wong "Unconditionally Stable Algorithms For Rigid Body Dynamics That Exactly Preserve Energy And Momentum", *International Journal For Numerical Methods In Engineering*, Vol. 30 (1990).

[106] Debra Lewis & J.C. Simo "Nonlinear Stability Of Rotating Pseudo-Rigid Bodies", *Proc. R. Soc. Lond.* A 427, pp. 281-319 (1990).

[107] J.E. Marsden, J.C. Simo, D. Lewis & T.A. Posbergh "Block Diagonalization And The Energy-Momentum Method", *Contemporary Mathematics*, Vol. 97 (1989).

[108] D. Lewis & J.C. Simo "A Comparison Of Non-Linear Stability Analyses By Energy Methods", To appear *Proc. 6th Symposium on Continuum Models and Discrete Systems.*

[109] D. Lewis, J.E. Marsden, T.S. Ratiu & J.C. Simo "Normalizing Connections And The Energy-Momentum Method", Technical Report, *Center for Pure and Applied Mathematics, University of California, Berkeley* (1990).

[110] J.C. Simo and N. Tarnow, "The Discrete Energy-Momentum Method. Part I. Conserving Algorithms for Nonlinear Elastodynamics," SUDAM Report No. 92-3, Stanford University, 1992.

[111] J.C.Simo and M.Doblare, "Nonlinear Dynamics of Three-Dimensional Rods: Momentum and Energy Conserving Algorithms," SUDAM Report No. 91-3, Stanford University, 1991.

[112] L-S. Wang, *Geometry, Dynamics and Control of Coupled Systems*, Ph.D. dissertation, Department of Electrical Engineering, University of Maryland, College Park, 1990. (also Systems Research Center Thesis Report *Ph.D. 90-5.*) (Advisor: P.S. Krishnaprasad)

[113] R. Yang, *Nonholonomic Geometry, Mechanics and Control*, Ph.D. dissertation, Department of Electrical Engineering, University of Maryland, College Park, 1992. (also Systems Research Center Thesis Report *Ph.D. 92-14.*) (Advisor: P.S. Krishnaprasad)

3. Specific Accomplishments of 1990-1994.

The work of the C-MULTICS project is characterized by collaborative effort between several of the team members and researchers at Maryland and other institutions. P.S. Krishnaprasad continued to collaborate with J.E. Marsden, A.M. Bloch, T.S. Ratiu as well as his students on problems of analytical mechanics and control. There has been extensive joint effort involving J.H. Maddocks, P.S. Krishnaprasad and L-S. Wang. The work of Stuart Antman has also involved collaborators (R. Marlow, J. Pierce, T. Seidman and M. Lanza de Cristoforis). In addition to his joint work with doctoral students, Eyad Abed developed a strong collaboration with Aerospace Industry (GE, Northrop, United Technologies) and Juan Simo has engaged in joint research with the group. Special efforts

have been made to understand the problems of interest to Air Force Laboratories and we have launched studies in several directions based on discussions with Air Force Laboratory personnel. In the following sections we report on our accomplishments in various categories along the lines of the original proposal. The numbers in parentheses refer to the list of papers in section 2.

The work of the project resulted in 8 Ph.D. dissertations (see references (4), (11), (20), (43), (91), (92), (112), (113), above) and 4 M.S. theses (see references (44), (45), (65), (66), above). Additional dissertations and theses are expected to be ready within the next six months. Five of the Ph.D. recepients hold tenure-track faculty positions and the rest are in research positions in government and industry.

3.1 Analytical Mechanics and Modeling

We made very significant progress in this area, covering much ground, ranging from new theory and formulations for shells to problems of fluid-structure interactions to deeper issues of bifurcation theory and material behavior to the hamiltonian structure of nonlinear control theory for broad classes of mechanical systems.

Paper (25) develops new methods of bifurcation theory to deal with the large buckling of nonlinearly elastic anisotropic plates. The anisotropy produces singularities in the equations that result in unexpected effects. Paper (26) combines local singularity theory and global bifurcation theory to show that compressible columns have surprising qualitatively different global behavior of solutions. Paper (27) shows how it is possible to obtain detailed qualitative information about buckled states of plates that can suffer thickness changes in addition to flexure, extension, and shear.

Papers (28), (29), (30), (33), (34), treat the deformation of nonlinearly elastic structures in perfect flows. A major goal of these analyses is to develop effective mathematical tools to handle nonlinear fluid-solid interactions in which the fluid dynamics is treated correctly. These papers involves very delicate constructions to determine requisite detailed information about the pressure field, which is not necessary for flow about rigid obstacles. Paper (34) treats a surprising instability that can occur when boundary conditions of a structure are subject to a feedback control that is designed to prevent unpleasant singularities in the flow from interacting with the structure.

Papers (31) and (35) show that refinements of traditional theories of columns and arches, inspired by the three-dimensional theory, lead to a far richer and more satisfying class of problems, which exhibit new kinds of important instabilities. Paper (32) shows that the traditional theory of internal constraints in continuum mechanics is too simplistic, because it ignores the fact that deformations are subject not only to pointwise contraints, but also to the compatibility conditions. A global constraint principle is introduced. Accordingly, the study of constraints and their multipliers requires analysis in function spaces, rather than in finite-dimensional spaces. The general theory resolves certain long-standing paradoxes in rod and shell theories.

Paper (37) gives a set of natural constitutive assumptions that ensure that a nonlinearly viscoelastic rod undergoing purely longitudinal motions cannot suffer a total compression at any point. It is perhaps surprising that the treatment of rods with the positions

of each end prescribed is far more delicate than those for rods with a force prescribed at at least one end. The resulting a priori estimates are crucial to the basic existence theory. This existence theory is fundamental for all theoretical and qualtitative studies of viscoelastic structures (in which the stress depends on the strain and strain rate). In particular, the ideas are central to the rigorous treatment of the problem of paper (39). Paper (37) is the only paper that treats longitudinal motions in which there is a physically natural nonlinear dependence on the strain rate. It exploits ideas that were first used in an earlier paper, which describes dissipative mechanisms for springs (Quart. Appl. Math, vol. 46, 1988, pp. 569-581).

The asymptotic limit of the equations of motion for a mass point attached to a sprir regarded as a continuum as ratio of the spings' mass to that of the mass point goes zero can differ markedly from that of the tradition model of a massless spring. Indeed, th leading term of the asymptotic limit typically describes a material with memory even when there is no memory in the original model. In this connection, a key contribution is paper (38), which treats the reduced asymptotic problem for the motion in space of a heavy rigid body on a light viscoelastic rod, which can suffer flexure, torsion, extension, and shear. It is shown that a general class of physically reasonable constitutive hypotheses allow the nonstandard reduced problem to be completely solved. The solution has a character far different from that for the purely longitudinal motion, which was treated a couple of years ago. The results throw light on the important issue of snapping motions. The paper is complete except for the preparation of illustrative graphs. A brief account of the contents appears in paper (39).

Book (40) contains a significant amount of new research along several of the areas described above.

Several research projects growing out from these are being pursued. Among these are (i) studies of asymptotics for light structures, which employ in a critical way the ideas of papers (32), (38),(ii) studies of dynamic bifurcations oriented toward fluid solid interations, which unite the work of these papers with papers (28), (29), (30), (33), (34), and (iii) studies of the control of smart materials.

In earlier work of Simo, Marsden and Krishnaprasad we completed our program on the systematic study of the hamiltonian structure of nonlinear elasticity, rods and shells. These results underscore the crucial role played by a convective description of elasticity, rods and shells and play a crucial role in the formulation of numerical algorithms as well.

More recently, methods of hamiltonian mechanics, stability theory and bifurcation theory play a central role in our work.

In (94), Maddocks obtains a comprehensive approach to the analysis of the second-order tests that determine which extremals of multiply constrained isoperimetric variational principles are actually constrained minima. The results are couched in terms of the shape, more specifically the singularities and curvatures, of the bifurcation hyper-surface that is obtained when a modified Lagrangian (evaluated on solutions) is plotted against the Lagrange multipliers associated with the constraints.

Many important Hamiltonian systems have periodic solutions that are associated with symmetries of the equations. While it is well known that stationary solutions of a Hamiltonian system can be characterized as extremals of the potential energy, it is less widely appreciated that symmetry-related periodic solutions, or relative equilibria, can also be given a variational characterization, typically involving constraints. This variational characterization is important because if a periodic solution is associated with a constrained minimizer (in some sense), as opposed to merelybeing a stationary point, then a stability result is very often available. We are therefore left with the problem of characterizing those extremals of a constrained variational principle that are actually constrained local minima. In (93) Maddocks shows how to apply the new results described in (94) in the special context of Hamiltonian mechanics, and various stability and instability theorems are described. The machinery developed here can be viewed as an alternative to the energy-casimir and energy-momentum methods.

In (96), the authors apply the variational bifurcation theory previously developed by Maddocks to consider the problem of shear-band formation in a thermo-plastic material where the constitutive law is dependent upon temperature and strain rate only. The existence of solutions and the qualitative form of the bifurcation diagram is obtained via a phase-plane analysis. Then stability properties of the solutions immediately follow from previous work on stability exchange.

In the joint work of P.S. Krishnaprasad, L-S. Wang and J.H. Maddocks (55), (58), the interactions of the attitude dynamics of large orbiting space structures with the central (earth's) garvitational field has been analyzed from a rigorous hamiltonian viewpoint. The methods of symmetry and reduction have been used to generate systematic approximations to the models of such systems. Hitherto unknown series of non-great-circle relative equilibria have been identified. Stability of certain great-circle relative equilibria has been investigated. This work is expected to be of importance to the accurate prediction of orbits of large earth-orbiting space platforms. Related work on tethered satellites appears in work of Liaw and Abed (3), (4), (7).

In the paper (58) which continues the analysis of (55) the authors rigorously prove the existence of non-great circle relative equilibria, which are families of relative equilibria that are qualitatively different from those previously known. These non-great circle relative equilibria arise for the exact potential of a finite size satellite, and their special properties are destroyed in the classic approximate analysis. We also show through numerical studies of the problem in the parameter regime pertinent to artificial satellites that differences between solutions to the approximate and exact models can be of physical significance. These studies lead to extremely stiff, numerically ill-conditioned systems, and associated difficulties which were resolved by performing computations on a CRAY super-computer to obtain additional accuracy (24 significant digits).

Paper (98) uses a variational principle associated with minimal frictional dissipation to analyze a problem that arises in some robot manipulation problems, namely the friction dominated motion of an object being pushed over a rough plane. It is shown that in certain physically plausible circumstances the problem for an arbitrary pressure distribution can be reduced to the case of singular pressure distributions with two-point support in the

boundary of the work piece, and this reduction is used to obtain calculable bounds on all possible motions.

The method of local Lie symmetry groups is used in (97) to systematically find all similarity solutions of the system of nonlinear partial differential equations that govern the quasi-static motion of a heavy chain being dragged over a rough table-top. The similarity solutions that we find include the shape of a steadily rotating segment with vanishing tension at one end, and a moving segment propagating into stationary chain. The problem presents interesting mathematical challenges because of the unusual form of the nonlinearity. The exact form of this nonlinearity is ubiquitous throughout models involving Coulomb friction and the chain is of some interest because it is the simplest example of a flexible body moving under the effects of friction.

In (95), (100), the authors use the approach of constrained variational principles to investigate the stability of the n-soliton of the Korteweg-de Vries equation. The theory leads to a stability result by showing that in the classic variational characterization of multi-solitons of KdV due to Kruskal and Lax, the multi-soliton actually realizes a constrained minimum (as opposed to any other type of critical point). This result extends the seminal work of Benjamin which considered the single soliton case $n = 1$. The methods of (100) promise to extend to various other classic partial differential equations with multi-solitons, as well as to other physically interesting systems such as the Toda lattice.

Abed has undertaken a deep study of issues of engine stall and compressor stall via computational bifurcation methods. In the collaborative work (18), for the first time, the nonlinear mechanisms of stall are beginning to be understood. Adomaitis, whos conducted some of these studies as a post- doctoral fellow, has pursued these investigations as a part of another AFOSR program as well as in this project as an Assistant Research Scientist.

P.S. Krishnaprasad and Rui Yang have completed their study of the dynamics of certain closed kinematic chains (69). This study provides a comprehensive picture of the role of modern dynamical systems methods in the understanding of mechanical linkages with constraints.

In the joint work of P.S. Krishnaprasad, L-S. Wang and W.P. Dayawansa (63), it has been shown that under certain resonance conditions, the eventual motions of mechanical systems with damping can include nontrivial (i.e. not relative equilibria) motions. This paper uses a combination of sophisticated geometric analysis and hands-on mechanical balance arguments for specific examples.

3.2 Nonlinear and Distributed Control

Our contributions in this area fall into several categories. Fundamental advances have been made in stability and instability theories. Some of this work has found its way into the feedback design and investigation of related stability issue in gyroscopic systems. Significant progress has been made in bifurcation control strategies. These have been applied in a variety of contexts, ranging from tethered satellite control, to high angle-of-attack flight control, to compressor stall control. In addition to advances in robust stability theory, new algorithms have been devised for stabilizing distributed parameter systems using complex variable techniques.

A key advance in stability analysis is the refinement of the energy-momentum method. In the recent work of L-S. Wang and P.S. Krishnaprasad (57), the block-diagonalization theorem has been generalized to gyroscopic systems with symmetry. This is independent of the related work of D. Lewis. Thus it has become feasible to prove results on closed-loop stability of spacecraft systems with internal reaction wheel control systems. These methods are quite capable of treating flexible and rigid systems. This work paves the way for a fundamental synthesis of the methods of geometric mechanics and those of geometric control theory.

A principal tool in this work is the gyroscopic generalization of the amended potential of Smale. This function carries a great deal of information regarding the relative equilibria, their stability and when parametrized by the "gyromomentum", also information on the bifurcations of relative equilibria.

In the work (56), of Bloch, Krishnaprasad, Marsden and Sanchez-de-Alvarez, energy-Casimir methods and geometric phases have been put to work to stabilization problems for rigid spacecraft.

One of the most challenging problems in large earth-observation spacecraft arises from the 'day-night' thermal cycling. As the satellite alternates between illumination by the sun and the darkness due to earth-occlusion, there are significant temperature changes in the flexible components of the spacecraft. These periodic changes force thermo-elastically driven vibrations which in turn cause attitude drift. A model problem of this type is the one on Figure 1 representing a rigid body with two driven mass-spring oscillators. This problem is analyzed carefully in the paper (64) and a simpler version is discussed in (60). Some related optimal control problems are explored and drift formulas presented.

Dissipation can induce instabilities. A powerful new theorem along these lines generalizing the classical work of Kelvin and Cetaev to relative equilibria has been proved by Bloch, Krishnaprasad, Marsden and Ratiu (62), (70). A major generalization of these results has also been submitted for publication (72). This result is expected to have very wide applications including the liquid-slosh problem referred to above. A finite dimensional example shows the possibility of instabilities due to actuator failure in a three axis stabilized satellite.

P.S. Krishnaprasad and his students have also undertaken an experimental program to explore actuator nonlinearities and impact problems. The M.S. theses (65), (66) and the papers (80), (81), (86) contain new results in friction modeling, friction compensation and impact force control.

Nonlinear phenomena can and do occur in space structures as well as in other engineering systems. Often these phenomena arise subsequent to parameter changes and can be viewed in terms of bifurcation theory. One goal of this research was to develop techniques for the feedback control of such phenomena in general nonlinear dynamical systems.

A significant body of results (due to Abed, his students, and collaborators) on control of systems undergoing static and dynamic bifurcations has resulted from the research effort. These are summarized in references (5), (6), (9), (10), (11), (12), (14), (15), (16),

(17), (19), (20), (21), (23), (24). Among these are two Ph.D. dissertations (11), (20). These results include the introduction of new paradigms in control of systems undergoing bifurcation and chaos, algorithms for stabilization of such systems, Liapunov functions for critical nonlinear systems, and new results on nonlinear stabilization with a prescribed domain of attraction. The application of these tools to control of chaos was made as the literature on problems of control of chaos was initiated and grew rapidly. The techniques developed under this grant have made a significant impact in the literature on control of nonlinear dynamics.

Tethered satellite systems are known to exhibit oscillatory behavior. Such behavior occurs in each of the station-keeping, deployment, and retrieval modes of operation. One goal of this research was to analyze the nature of this oscillatory dynamics and to determine feedback control laws for damping the oscillations.

Tethered satellite system stability and stabilization during station-keeping, deployment and retrieval were considered in the publications (3), (4), (7). Among these, (4) is a Ph.D. dissertation. New insights into this important problem were obtained. Nonlinear control was shown to be very important to control of the out-of-plane dynamics of tethered satellite systems. Specific nonlinear controllers were given and their stabilizing nature was demonstrated through rigorous analysis and simulation. New methods were given for Liapunov stability analysis of deployment and retrieval. Center manifold-based control design and normal forms were also employed to show the benefit of nonlinear control over linear control in this application.

Two important and rather natural applications of the bifurcation control approach are the control of stall in aircraft flight and in jet engines.

The bifurcation control techniques developed were employed in control of aircraft at high angles-of-attack and jet engine compressor systems. The former work is reported in references (2), (10), and (11), while the latter is reported in (13), (18), (20), and (22). Of these, references (11) and (20) are Ph.D. dissertations. Bifurcation control was used to result in proof-of-concept control laws for a model of an F-8 aircraft in the near-stall flight regime. Bifurcation analysis and detailed modeling was employed to obtain important new insights into the dynamics of stall phenomena in compressors. New insights were also obtained into the control of these stall phenomena using bifurcation control techniques. The latter work was in close collaboration with General Electric.

In our program of investigation into robust stability, the research effort concerned determination of conditions for stability of linear systems depending on parameters. This is in synergy with the other aspects of the project, since loss of stability and bifurcation of nonlinear systems operating at nominal equilibrium points is closely linked to loss of stability of their linearization. Robust stability results obtained under this grant include the introduction of the concept of guardian map and its use. This work is reported in references (1) and (8).

In paper (42) experimental results on parameter identification for an in-orbit satellite are discussed. The vehicle was the LACE (Low Atmospheric Experimental Control

Experiment) spacecraft. This low-earth orbit satellite occupies a circular orbit. Its structure consists of a central rigid body or bus with three deployable booms. One of these booms is the gravity gradient boom, and the other carries the retro-reflector. Identification of structural modes was accomplished in-orbit via ground-based laser illumination with Doppler-shifted returns. Frequency-domain and time-domain error bounds were obtained. The experiment was performed in conjunction with the Naval Research Laboratory. Details of the data analysis are to be found in the M.S. thesis (44) of D. Augenstein. More recent identification results are to be found in (81), (83), (84).

The Ph.D. dissertation of El-Baraka (43) discusses his results on the problem of optimal state feedback control for flexible structures using a spectral factorization method. The work considers examples arising from one dimensional and two dimensional structures, including strings, membranes and Euler-Bernoulli beams with boundary control. This work shows the computational power of spectral factorization methods. In the work of Berenstein and collaborators, the main mathematical idea considered has been to make as explicit as possible the solutions $g_1, \cdots, g_m$ of the Bezout equations:

$$\text{(BE)} \qquad f_1 g_1 + \cdots + f_m g_m = 1$$

(i) $f_1, \cdots, f_m$ are entire functions of n complex variables with growth conditions, typically Paley-Wiener class.
(ii) $f_1, \cdots, f_m$ are polynomials of n variables.

A key idea has been that the complex analysis tools developed by Berenstein and Yger (48) to study (i) should also be useful to consider (ii). They considered a novel way of using integral representation formulas to solve the Cauchy-Riemann equations, in particular, by recognizing the fundamental role of multidimensional residues. This has lead to the use of very powerful ideas of D-modules and analytic continuation of powers to obtain more explicit solutions of (BE) with better estimates.

The paper (52) summarizes the impact of these techniques for the algebraic Bezout equation, and gives a complete picture of the known estimates in complexity of solvability, estimates of degrees and sizes of coefficients, of polynomial solutions $g_1, \cdots, g_m$ of (BE). Some of these estimates are due to Yger and Berenstein (46), some to Brownawell, Kolar, Canny, Rennegar, Heintz.

The Bezout Equation (BE) appears in the context of the present project in the design of compensators for systems governed by partial differential equations. Bernard Frankpitt, a joint student of J. Baras and C. Berenstein, has used (45) the above methods to study some concrete control problems. In his work on the control of a beam, when one allows for some delay to occur between the measurement and the action of the controller, one is lead to (BE) with analytic coefficients, which we can try to solve in H^∞ or in the Paley-Wiener class. Frankpitt has made some progress using the methods developed by Yger and Berenstein to solve (BE) in that case. One item of note is that, in fact, in the examples considered by Frankpitt the functions f_j are very special, they are exponential-polynomials.

At the Intelligent Servosystems Laboratory, some of these design methods have been integrated into an object-oriented optimization based design framework. Real-time control studies have also been carried out. A new real-time control design environment has been acquired.

Extensive new work on nonholonomic mechanics and control has been carried out in the last two years (77), (74), (73), (87)-(91), (113). Some of this work has led to new constructive control algorithms.

3.3 Advanced Computation Research

We have made significant progress in developing new algorithms and software tools for modeling, simulation and control design. Paper (110) is a major contribution to the numerical integration of elastodynamics. In the absence of external loads or in the presence of symmetries (e.g. rigid motion group invariance), the nonlinear dynamics of continuum systems preserve the total linear and angular momentum. Furthermore, under assumption met by all classical models, the internal dissipation is non-negative. In (110), Simo and Tarnow present systematic design of conserving algorithms that preserve exactly the conservation laws of momentum and inherit the property of positive dissipation for any step-size. In particular within the context of elastodynamics, a second order accurate algorithm is presented that exhibits exact conservation of both linear and angular momentum, and of total energy. This scheme is amenable to a completely straightforward (Galerkin) finite element implementation and ideally suited to long-term/large scale simulations. The excellent performance of the method relative to conventional time-integrators is conclusively demonstrated in numerical simulations exhibiting large strains coupled with large overall rigid motion.

Paper (111) of Simo and Doblare is concerned with accurate integration of 3-d geometrically exact rod dynamics undergoing shear, finite extension, bending and large overall motions. Here the preservation of momentum maps is shown to be achieved for only a special class of algorithms, and depends critically on a formulation of the equations of motion in conservation (or impulse) form. This class of conserving algorithms exhibits no numerical dissipation. A number of numerical simulations confirm the unconditional stability and conservation properties predicted by the theoretical analysis.

Complementing this approach, Austin, Krishnaprasad and Wang (61),(68), have developed techniques based on the midpoint rule for a wide variety of rigid body systems that admit Lie-Poisson structures. This work presents an error formula for the Poisson structure, the idea being that one uses an almost Poisson algorithm but conserves energy and momentum exactly. These algorithms have been implemented in the Intelligent Servosystems Laboratory in a distributed computational environment.

Higher order schemes based on extrapolation have been developed by Austin in his ongoing work. We hope to investigate similar methods on Lie groups.

3.4 Laboratory Development

The Intelligent Servosystems at the University of Maryland, College Park, has been thriving as a facility for experimental and computational research. The laboratory has

a network of hP workstations including a TurboSRX with 3D graphics, a network of Silicon Graphics workstations and assorted Sparc-stations, and other computers. The experimental facilities include a flexible arm with electrical slip ring to study smart actuator based stabilization of vibrations.

We are also continuing our studies in the real-time control of flexible arms including force and impact control issues.

3.5 Visitor Program & Air Force Lab Interactions

Recent visitors to the University of Maryland interacting with AFOSR-URI project participants include Professor Anthony Bloch from Ohio State, Professor Dave Gilliam from Texas Tech, Professor Clyde Martin from Texas Tech, Dr. James Gillis from the Aerospace Corporation, Dr. Venkataraman from JPL, Professor Jerrold Marsden of UC Berkeley, Professor of Shankar Sastry of Harvard, Professor Roger Brockett of Harvard, and various industrial scientists.

During the summer of 1991, several visits and exchanges with Air Force Labs were held. The following is a set of trip reports from these visits. Additional trips were made to Wright Patterson Air Force Base (Fall 93), General Electric (Spring 1994).

REPORT ON A TRIP TO EDWARDS AIR FORCE BASE, Phillips Laboratory
(to facilitate transitions from the URI program to Air Force Laboratory Programs.)

Dr. P. S. Krishnaprasad visited the Structures and Control Branch of Phillips Laboratory at Edwards Air Force Base on August 12, 1991. This visit was planned with the help of Dr. Marc Q. Jacobs, Program Manager, Mathematics of Dynamics and Control, AFOSR, and was funded through the AFOSR University Research Initiative (URI) Grant at the Systems Research Center, University of Maryland. Mr. Kevin Slimak and Mr. Monty Smith of Phillips Laboratory are to be thanked for coordinating this visit.

The purpose of this brief write-up is to document the visit and potential areas of collaboration between the Maryland URI team and engineers at Phillips Laboratory. We are pleased to note some areas for immediate transition, and some for future collaboration.

Schedule: (Monday, August 12, 1991)

Seminar: Overview of AFOSR-URI Project on Control of Complex Multibody Spacecraft by P.S.Krishnaprasad, at Aries Conference Room, 10:00 - 11:30 a.m.

Lunch Break: 11:30: - 12:30.

Discussions on MULTIBODY Project at Phillips Lab: 12:30 - 13:30 with Monty Smith

Lab Tour: 13:30 - 15:00 , arranged by Monty Smith. Facilities visited include (a) ASTREX facility, (b) MULTIBODY project site, (c) Composites Lab and smart sensors and actuators experiments. Also, discussions with Joel Berg, Lt. Steve Griffin.

Discussions: 15:00-15:30, Distributed parameter control and actuator modeling with Monty Smith.

Closing: 15:30 - 15:40, Kevin Slimak.

Two main topics for discussions were the MULTIBODY project at Phillips and the role of active and passive damping in structures especially the type of smart members to be supplied by TRW for inclusion in the ASTREX test article. The MULTIBODY project is also designed as a test article to explore control laws, issues in model validation, actuator modeling including bearing friction in motors, piezoelectric actuation for damping of links etc. The various parts for the project are in hand and the assembly is expected to be completed by the end of 1991. The ASTREX facility is very nearly completed and is expected to be available for guest investigators.

(1) MULTIBODY PROJECT:

The experimental setup when completed will consist of two planar flexible links actuated with two motors, one supported by air bearing on a very flat granite table. The links

will be of fiberglass and one of the two PMI D.C. motors will be inertially fixed. Data acquisition and real-time control support will be provided by a CDAC crate similar to the one used for the ASTREX. A laser position measurement system will be used to provide absolute position data for evaluation. The links, each a meter long, will have a 0.5 Hz first mode. Among the various tasks at hand, we note accurate modeling of motors (friction, torque ripple etc.), Galerkin approximations for the full system of coupled rigid and flexible bodies, identification of friction using deterministic and stochastic adaptive methods, creation of a comprehensive simulation model for comparison against experimental results, and for validation of the methods of formulating the models. This experiment should serve as a very useful testbed for a variety of modeling and control applications.

Action item (1)

There are some significant similarities with the single link experimental flexible arm in the Intelligent Servosystems Laboratory at Maryland. The work done in Maryland on friction compensation using adpative control techniques should also be useful to the MULTIBODY PROJECT. Under the supervision of P.S. Krishnaprasad, a simulation model will be created at Maryland using existing software tools and it will be made available to Phillips laboratory. Additional studies will be conducted once real-time data becomes available. Also at Maryland, analytical and experimental studies will be initiated to explore the use of piezo-electric damping layers for use in the fiberglass links. Other actuation material such as Terfenol will also be considered.

(2) ASTREX TEST ARTICLE

This very substantial testbed will serve as a useful vehicle for the study of control-structure interactions. Based on discussions during the lab tour, it appears that it would very useful to investigate a pointing and tracking problem in the presence of gas jets and a single reaction wheel. Some nonlinear control theoretic ideas should be useful in this connection. In addition, a deeper understanding of the design problem for piezo actuated truss members (e.g. the TRW truss member) is needed. At present the main control design ideas seem to revolve around the use of notch filters and local analog/hybrid servo loops. The URI project should be able to contribute to the theoretical basis for such problems.

(3) COMPOSITES LAB/SMART ACTUATORS

A very good technical discussion ensued at the composites lab with Lt. Steve Griffin on the problem of designing circuits for piezo actuators. There was also a good discussion on the problem of controller design with Monty Smith. It appears that further theoretical advances are needed to exploit piezo actuators in a broad band fashion.

Action Item (2)

The URI team will contribute some further analysis, based on the research literature, of the piezo actuator problem. In particular, work will be done on the problem of state space modeling of the flexible member with embedded piezo material.

POSSIBLE TRANSITIONS

It appears that immediate transitions are possible in the modeling and control problem for the MULTIBODY PROJECT at Phillips Lab (see action item (1) above.). Some of this may take the form of software tools and some the form of numerical simulation results. Further transitions are expected in the ASTREX project and in the area of piezo actuators.

REPORT ON A TRIP TO EGLIN AIR FORCE BASE

(to facilitate transitions from the URI program to Air Force Laboratory Programs.)

Dr. Eyad H. Abed visited the Flight Control Technology Section (MNAG) of Eglin AFB on June 13 and 14, 1991. This visit was planned with the help of Dr. Marc Q. Jacobs, Program Manager, Mathematics of Dynamics and Control, AFOSR, and was funded through the AFOSR University Research Initiative (URI) Grant at the Systems Research Center, University of Maryland directed by Professor P.S. Krishnaprasad. Mr. Johnny Evers (MNAG) is to be thanked for his help in coordinating the visit.

The purpose of this brief write-up is to document the visit and potential areas of collaboration between the Maryland URI team and engineers at Eglin. We feel that there are clear areas of possible fruitful collaboration, some of which are identified below. Other possibilities may arise in the course of future discussions.

The schedule for the visit, as announced at Eglin prior to the visit, was as follows:

Thursday 13 June Bldg 1, Rm 204

8:00-8:30 Introductions
8:30-9:00 Overview of Armament Directorate — Govt
9:00-10:00 Some current nonlinear control problems and tools — Abed

- Stabilization and tracking
- Geometric control
- Liapunov-based methods
- Bifurcation and chaos in control

10:00-10:30 Break/discussions
10:30-11:30 Bifurcations - Theory and computation — Abed

- Local bifurcations
- Global bifurcations
- Examples (Compressors, Aircraft, etc.)

11:30-1:00 Lunch
1:00-2:00 Control and stabilization of bifurcations — Abed

- Goals of control
- Mathematical Tools
- Local stabilization
- Washout filters

2:00-2:30 Break/discussions

2:30-3:30 Applications of bifurcation control — Abed

- Tethered satellites
- High alpha flight control
- Surge/stall control of compressors
- Control of a chaotic system

Friday 14 June Bldg 13, Rm 127

8:30-9:30 Missile Flight Control Systems — Govt
9:30-10:00 Break/Discussion
10:00-11:00 Recent developments in control theory for missile autopilots — Govt
11:00-11:30 Discussion
11:30-1:00 Lunch

As the schedule above indicates, Dr. Abed gave four one-hour presentations on Thursday June 13. The first of these was a general overview of nonlinear control, and the remaining three were on specific topics that Dr. Abed has worked on extensively. Personnel from Eglin AFB were invited to attend any or all of the presentations on Thursday June 13 that interested them. The audience consisted of representatives of several different organizations at Eglin, namely WL/MNAG, MNAV, WL/MNGA, WL/MNMF, MNSI, MNAA, and Svd/TEAS. At its peak, the attendance consisted of eighteen individuals. The seminars resulted in a large number of questions from the audience, and there was a general feeling that this was a useful educational experience. The first presentation, gained the appreciation of a number of the attendees, since it allowed them for the first time to gain some understanding of basic issues and techniques of geometric control.

The Friday session was intended to familiarize Dr. Abed with Air Force control problems. This session took place in the offices of the Flight Control Technology Section. It consisted of informal presentations led by Eglin engineers regarding current and future problems being considered at Eglin in flight control systems and related topics.

Among the technical issues discussed on June 14 were:

- Release of a new missile simulation/control model to the University of Maryland upon its completion (approx. 1 year) for collaborative research on guidance laws
- Importance of research in actuator saturation related problems to ongoing efforts at Eglin
- Eigenstructure assignment issues as they relate to a roll/sideslip decoupling problem for missiles
- Alternative analytical and numerical tools that might prove useful in a variety of missile guidance and control problems (CONSOLE, "participation factors," geometric control, etc., all areas in which the Maryland URI faculty has expertise)

Without going into the technical details of these issues, we can summarize that this visit was successful and beneficial to both Maryland and Eglin. Further contacts with Eglin engineers are continuing. We are providing references which we feel could be useful in Air

Force control problems, and, at a later stage, will pursue actual joint research incorporating our ideas in the control problems provided by the Flight Control Technology Section, MNAG.

Participants from Eglin AFB in Seminars/Discussions

Name	Organization	Also participated 6-14-91 (*)
Jim Cloutier	MNAG	*
Chris D'Souza	MNAG	*
Johnny Evers	MNAG	*
Debra Harto	MNAG	*
Corey Schumacher	MNAG	*
Darren Schumacher	MNAG	*
Robert F. Wilson	MNAG	*
Randy Zachery	MNAG	*
Jack George	MNAA	
Kirk Vanden	MNAA	
Tom Barth	MNAV	
Charlesworth Martin	MNGA	*
Martin J. Moorman	MNGA	*
Hisook Min	MNMF	
Kwang Min	MNMF	
W. Zimmermann	MNMF	
Steve Perillo	MNSI	
Bernie Asner	Svd/TEAS	

REPORT ON A TRIP TO JET PROPULSION LABORATORY

(to facilitate transitions from the URI program to Air Force Laboratory Programs.)

Dr. P. S. Krishnaprasad visited the Machine Intelligence Applications Group at the Jet Propulsion Laboratory and presented a JPL seminar. This visit was funded through the AFOSR University Research Initiative (URI) Grant at the Systems Research Center, University of Maryland, College Park. Dr. Guillermo Rodriguez was the host and deserves special thanks for making this a most enjoyable visit.

The purpose of this brief write-up is to document the visit and potential areas of collaboration between the Maryland URI team and engineers at JPL. We are pleased to note some areas for immediate transition, and some for future collaboration. It is important to note that there is a close involvement of some of the personnel in the Machine Intelligence Applications Group and some of the researchers in the Control-Structures Interactions Group who are engaged in Air Force Funded Projects.

Schedule:	(Tuesday, August 13, 1991)
9:30- 10:30	Discussions with Dr. Guillermo Rodriguez, Room 301-127
10:30- 12:00	Seminar: Nonholonomic Geometry, Mechanical Systems & Optimal Control, Room 301-207 (12 attendees).
12:00- 13:00	Lunch Meeting with Dr. A.K. Bejczy
13:00- 13:30	Discussions on Multibody Dynamics with Dr. A. Jain
13:30- 14:30	Lab Tour with Mark Long (discussions on autonomous systems and software aspects)
14:30- 15:00	Lab Tour with Richard Volpe (discussions on on-orbit inspection systems and on impact dynamics and control)
15:00- 15:30	Discussions on real time control and sensing with Zoltan Szakaly
15:30- 16:30	Discussions on Nonholonomic mechanics with Jonathan Cameron
16:30- 17:30	Closing discussions (on multibody dynamics, collaborative research, summer students etc.) with Dr. Rodriguez.

Two main topics for discussions were multibody dynamics formulations of O(n) complexity and the role of modern nonlinear analysis methods to solve design and control problems associated to space-based articulated structures. Examples of interest arose from problems due to nonlinear oscillations (possibly induced by stiction effects) in space-based flexible manipulators. Additional concerns include gear-backlash effects and the role of active and passive damping in structures especially the type of smart members to be supplied by TRW for inclusion in the ASTREX test article at Phillips laboratory. While many of

these concerns pertain to NASA missions such as CRAF-CASSINI, the basic problems that have been investigated under the URI project are of direct interest to people involved at JPL in such missions and flight systems software. Future interactions on control-structure problems are expected with people working under Air Force funded projects at JPL.

TRANSITIONS

There is considerable interest in possible use of the software and analysis tools developed under the URI project. The inter-process communications software developed by Russell Byrne in his M.S. thesis at Maryland has been installed on a network in section 366 (Advanced Information Systems Section, in the High Speed Distributed Systems Group). Mr. Byrne, a former advisee of P.S. Krishnaprasad, now works for section 366 and also with some people from the Hypercube Project, on an object-oriented, concurrent Air Defense Simulation for the USAF Electronic Systems Division.

Further collaboration is expected with Rodriguez on the problem of fast real-time multi-body simulation. It is possible to run trial experiments on the intel i860 board at the Intelligent Servosystems Laboratory at Maryland, identical the one recently acquired by JPL. This could lead to some real comparison of results.

There is considerable interest in JPL in our work on smart actuators, friction compensation and adaptive control. A very positive feeling of appreciation was noticed during this visit. This will be a basis for significant collaborative research. Since the visit, a proposal for support for a graduate fellowship has been submitted to JPL and is under review.

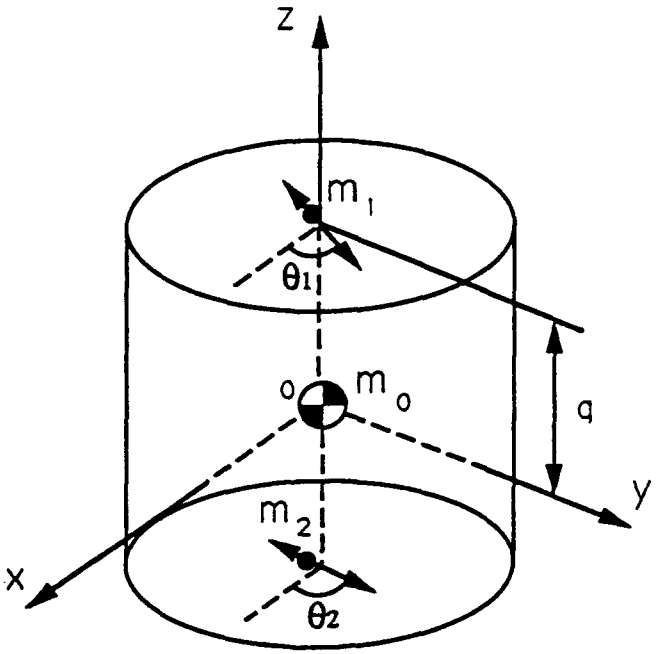

Figure 1

Body & Oscillators